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# MAXIMAL SPECIFICITY AND LAWLIKENESS IN PROBABILISTIC EXPLANATION*

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The article is a reappraisal of the requirement of maximal specificity (**RMS**) proposed by the author as a means of avoiding "ambiguity" in probabilistic explanation. The author argues that **RMS** is not, as he had held in one earlier publication, a rough substitute for the requirement of total evidence, but is independent of it and has quite a different rationale. A group of recent objections to **RMS** is answered by stressing that the statistical generalizations invoked in probabilistic explanations must be lawlike, and by arguing that predicates fit for occurrence in lawlike statistical probability statements must meet two conditions, at least one of which is violated in each of the counterexamples adduced in the objections. These considerations suggest the conception that probabilistic-statistical laws concern the long-run frequency of some characteristic within a reference class *as characterized by some particular "description"* or predicate expression, and that replacement of such a description by a coextensive one may turn a statement that is lawlike into another that is not. Finally, to repair a defect noted by Grandy, the author's earlier formulation of **RMS** is replaced by a modified version.

**1. The rationale of the requirement of maximal specificity.** In this article, I propose to reconsider certain basic issues in the logic of probabilistic-statistical explanation and to respond to some criticisms and constructive suggestions concerning my previous writings on the subject.

In my articles [4] and [5], I contrasted probabilistic, or inductive-statistical (I-S), explanation with deductive-nomological (D-N) explanation. A D-N explanation is an argument in which the explanandum sentence, which describes whatever is being explained, is deduced from a set of explanans sentences which include one or more laws or theoretical principles and usually, though not necessarily, also certain statements of particulars, such as initial or boundary conditions. An argument of this kind explains the explanandum phenomenon by showing that it was to be expected in view of the general laws adduced, given the particular circumstances specified. Such an account might, therefore, be said to exhibit the nomic expectability of the explanandum phenomenon.

A statistical explanation, too, relies on laws; but at least one of these is of a probabilistic-statistical character. The simplest laws of this kind have the form: 'the statistical probability for an $F$ to be a $G$ is $r$', or '$p(G, F) = r$' for short; they are the probabilistic counterparts of strictly general laws of the form 'All $F$ are $G$'. But while a law of the latter kind, combined with the particular statement '$i$ is $F$' deductively implies '$i$ is $G$' and thus affords a corresponding D-N explanation, the

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statistical law '$p(G, F) = r$' combined with '$i$ is $F$' can be said to explain $i$'s being $G$ only inductively, i.e. in the sense that it lends more or less strong inductive support to the explanandum sentence '$i$ is $G$'. For reasons indicated in [5], pp. 377–378, I took this inductive support numerically to equal to $r$, and I schematized the resulting I-S explanation thus:

$$(1) \qquad \frac{\begin{array}{l} p(G, F) = r \\ Fi \end{array}}{Gi} \qquad [r]$$

Explanatory arguments having this structure I called I-S explanations of basic form; and, as in my previous papers, I will limit my discussion here to the logic of this simplest type of probabilistic explanation. The number indicated in brackets is "the probability associated with the explanation"; it is not a statistical probability, but an inductive one in Carnap's sense, namely, the probability of the explanandum relative to the explanans. The argument *explains* $i$'s being $G$ by showing that this is to be expected, with probability $r$, in view of the general statistical law and the statement of particular fact included in the explanans. The argument will be considered as explanatory only if $r$ is sufficiently close to 1; but no specific common lower bound for $r$ can reasonably be imposed on all probabilistic explanations.

In offering an explanation of either kind for a given phenomenon, we claim of course not only that the argument in question is "valid"—that its "conclusion" bears the requisite logical relation to the "premisses"—but we also affirm the premisses: just as in making an assertion of the form '$B$ because $A$' we implicitly claim that $A$ is the case.

Explanations based on probabilistic-statistical laws may exhibit what I have called statistical ambiguity. Given an argument of type (1) in which both premisses are *true* and the associated probability $r$ is close to 1, there may exist a "conflicting" I-S argument

$$(2) \qquad \frac{\begin{array}{l} p(-G, H) = s \\ Hi \end{array}}{-Gi} \qquad [s]$$

which also has *true* premisses and an associated probability close to 1, and which therefore explains $i$'s not being $G$ just as the first account explains $i$'s being $G$.

For example, let $j$ be some particular individual, Jones; let '$Ix$' stand for '$x$ has been infected with malaria plasmodium'; '$Mx$' for '$x$ contracts malaria'; and '$Sx$' for '$x$ is a heterozygote in regard to the sickle hemoglobin gene', which means that $x$ has acquired that gene from one, but not both, of his parents. This characteristic $S$ has been found to afford strong protection against malaria.[1] Let us assume, to be specific, that $p(-M, S) = .95$ and $p(M, I) = .9$. Suppose now that Jones has been infected with malaria plasmodium, but has the protective characteristic $S$.

[1] See, for example, Glass [2], pp. 57–58.

Then the following two arguments have true premisses and thus form conflicting inductive-statistical accounts:

$$\text{(3a)} \quad \frac{\begin{array}{l} p(M, I) = .9 \\ Ij \end{array}}{Mj} \quad [.9]$$

$$\text{(3b)} \quad \frac{\begin{array}{l} p(-M, S) = .95 \\ Sj \end{array}}{-Mj} \quad [.95]$$

This possibility of "explaining" by true statements both the occurrence of a phenomenon and its nonoccurrence throws serious doubt on any claim to explanatory power that might be made for such inductive-statistical arguments. (D-N explanations are not subject to any such ambiguity: the existence of a true D-N explanans for the occurrence of a given phenomenon logically precludes the existence of a true D-N explanans for its nonoccurrence.)

As a way of bypassing (though not resolving) this ambiguity, I suggested that I-S explanation, in contrast to D-N explanation, be construed as an epistemological concept explicitly relativized with respect to a given "knowledge situation." The latter would be formally represented by a class $K$ containing all those sentences—whether actually true or false—which are accepted as presumably true by the person or persons in the given knowledge situation. By way of idealization, the class $K$ will be assumed to be logically consistent and closed under the relation of logical consequence, and to contain the theories of the statistical and the logico-inductive concepts of probability.

We are thus led to consider the concept of "I-S explanation relative to $K$ (or, in $K$)." If an argument of the form (1) is to qualify as such an explanation, its premisses will have to be in $K$.[2] But this condition alone cannot prevent explanatory ambiguity from recurring in a new variant: a class $K$—for example, the class of sentences accepted in contemporary science—may evidently contain the explanans sentences for two conflicting explanatory arguments, such as (3a) and (3b). This variant might be referred to as *epistemic ambiguity*, in contrast to the kind described first, which could be called *ontic ambiguity* of I-S explanation. The latter results from the existence of nomic and particular facts, or of corresponding true sentences—no matter whether known or believed—which give rise to I-S arguments with true premisses and logically incompatible conclusions; epistemic ambiguity results from the fact that the class $K$ of sentences believed or accepted in a given knowledge situation—no matter whether they are true or not—may similarly contain premiss-sets for incompatible conclusions. (D-N explanation, let it be noted in passing, cannot be epistemically ambiguous, any more than it can be ontically ambiguous.)

[2] In order also to accommodate explanations which are merely proposed or contemplated rather than asserted—i.e. whose explanans sentences are not, at least as yet, included in $K$—my earlier treatment of the subject made slightly more complicated provisions (cf. [5], p. 400, note 20); but the logically crucial points can be stated more simply if we require, as I do here, that the explanans sentences of any I-S explanation relative to $K$ must belong to $K$.

A simple and plausible way out suggests itself here. An explanation is normally asked for only when the explanandum phenomenon is taken to have occurred, i.e. when the explanandum sentence belongs to *K*. Suppose we require accordingly of any I-S explanation in *K* that its explanandum sentence belong to *K*: then the consistency of *K* precludes conflicting explanations like (3a) and (3b). I find the requirement a very reasonable one for a concept of explanation that refers explicitly to what is taken to be the case; and I will therefore adopt it. But though it never grants explanatory status to two arguments with logically incompatible premisses, it does not eliminate what seems to me the objectionable aspect of explanatory ambiguity. For in a case where *K* contains the premisses and the conclusion of (3a), as well as the premisses of (3b), we would still be able to say: "*i* is *G*, and we can explain that by (3a); but if *i* had turned out not to be *G*, our total knowledge would just as readily have afforded an explanation for that namely, (3b)." If, as I think, an explanation exhibits the strong nomic expectability of a phenomenon then surely that claim cannot hold good.

Should we, then, stipulate instead that an argument like (3a) qualifies as an I-S explanation relative to *K* only if *K* does not contain the premisses of any "conflicting" I-S argument, such as (3b)? This requirement would indeed bar explanatory ambiguities, but it is too restrictive. For suppose that *K* contains the premisses of (3a) and (3b) and also those of the argument

$$\text{(3c)}\qquad \begin{array}{l} p(-M, S \cdot I) = .95 \\ Sj \cdot Ij \\ \hline\hline -Mj \end{array} \qquad [.95]$$

Assuming that *K* contains no further statements—i.e. none that are not logically implied by those just specified—we would presumably say that (3c) afforded an I-S explanation, relative to *K*, of why Jones did not catch malaria. And we would grant (3c) this status even though our knowledge *K* also contains the premisses for the conflicting argument (3a). For while *K* does inform us that Jones belongs to the class *I*, and that within *I*, the feature *M* has the high statistical probability .9, *K* contains the further information that Jones also belongs to another class *S*, and hence to the class $S \cdot I$; and that, among the members of that subclass of *S*, *M* has the very low probability .05. Finally—and this is decisive—*K* does not assign Jones to a still narrower reference class; hence the I-S argument (3c) is based on the most specific information we have about Jones in the given knowledge situation; and that would seem to confer on (3c), but not on (3a), the status of an explanation relative to *K*. What of (3b)? Since, according to the first premiss of (3c), the statistical probability of not contracting malaria is the same in $S \cdot I$ as in *S*, the factor *I* is probabilistically irrelevant to $-M$ relative to *S*. For this reason, (3b), too, may count as an explanation of Jones's not contracting malaria.[3]

Considerations of this kind led me to propose, in [5], a "requirement of maximal

[3] For convenience of formulation, I permit myself here to speak of one-place predicates as standing for properties, features, or characteristics of objects or events, and alternatively as standing for the corresponding classes. An important *caveat* concerning this usage is noted in section 3 below.

specificity" intended to preclude explanatory ambiguity. In substance it provided that an argument

$$(4) \qquad \frac{\begin{array}{c} p(G, F) = r \\ Fi \end{array}}{Gi} \quad [r]$$

where $r$ is close to 1 and both premisses are contained in $K$, constitutes a probabilistic explanation relative to $K$ only if it meets the following condition:

(**RMS**) For any class $F_1$ for which $K$ contains statements to the effect that $F_1$ is a subclass of $F$ and that $F_1 i$, $K$ also contains a probabilistic-statistical law to the effect that $p(G, F_1) = r_1$, where $r_1 = r$ unless the law is a theorem of probability theory.[4]

The 'unless'-clause is meant to allow $K$ to contain, without prejudice to the explanatory status of (4), pairs of sentences such as '$Fi \cdot Gi$' and '$p(G, F \cdot G) = 1$'; the latter, being a theorem of the probability calculus, is thus not reckoned as an explanatory empirical law—just as '$(x)[(Fx \cdot Gx) \supset Gx]$' does not qualify as an explanatory law on which a D-N explanation can be based. Similar remarks apply to probability statements such as '$p(G, F \cdot -G) = 0$'.

In our example concerning Jones's malaria, the condition **RMS** is met by (3b) and (3c), but not by (3a), which is just the result we want.

**2. A clarification: Maximal specificity *vs.* total evidence.** A requirement to essentially the same effect as **RMS** was proposed already in my essay [4] (pp. 146–148), but on grounds which, I now think, misconstrued the relationship of **RMS** to the requirement of total evidence. I argued there that explanatory ambiguities like those arising for a class $K$ containing the premisses of both (3a) and (3b) are properly and readily avoided by heeding the requirement of total evidence, i.e. by assigning inductive probabilities to the conflicting explanandum sentences on the basis of the total evidence available in the given knowledge situation, that is, on the basis of the entire class $K$. This, after all, is a principle which, as stressed by Carnap and others, must be observed in all rational applications of probabilistic inference.[5] Now, whatever the class $K$ may be, it cannot confer high inductive probabilities on both of two contradictory explanandum sentences since their probabilities must add up to unity. Thus, I concluded, adherence to the requirement of total evidence is the way to avoid explanatory ambiguity.

But, I noted further, even if we assume that an adequate general definition of logical probability can be given—perhaps in the general manner proposed by Carnap—it would be a hopelessly complex task actually to compute the probabilities of two conflicting explanandum sentences with respect to the vast set $K$ representing our total putative knowledge. It would be desirable, therefore, to have

[4] For the reasons referred to in note 2, my formulation of the requirement in [5], p. 400, is slightly more involved; but the substance of the earlier version is essentially the same as that of **RMS** above.

[5] See [1], pp. 211–213.

a practicable method of assigning to those sentences at least approximations of their probabilities relative to $K$. And I suggested, in effect, that if $K$ contains the premisses of an argument of type (4) and satisfies the maximal specificity condition with respect to it, then the logical probability $r$ of '$Gi$' relative to the premisses of (4) may be considered as an approximation of the probability of '$Gi$' with respect to the total evidence $K$ ([4], pp. 146–147). In support of this suggestion, I offered some plausibility considerations for special cases; but I also noted, by reference to a specific example, that under certain conditions, the value of $r$ may be quite different from the probability of '$Gi$' on $K$ ([4], pp. 148–149). I therefore characterized my rule as only a "rough substitute for the requirement of total evidence" ([4], p. 146) and concluded that "the requirement of total evidence remains indispensable" for the assignment of probabilities to the conclusions of I-S explanations ([4], p. 149).

But this reasoning confounds two quite different questions. One of these concerns the strength of the evidence for the *assertion that* the explanandum event did occur; the other, the probability associated with an I-S *explanation of why* the explanandum event occurred. In reference to our simple schema (4), the first question might be put thus:

(5a) What degree of belief, or what probability, is it rational to assign to the statement '$Gi$' in a given knowledge situation?

Here, the requirement of total evidence applies. It directs that the probability should be determined by reference to the total evidence available, i.e. by reference to the entire class $K$.

The second question does not concern the grounds on which, and the degree to which, it is rational to believe that $i$ is $G$, but the grounds on which, and the strength with which, $i$'s being $G$ can be explained, or shown to be nomically expectable, in a given knowledge situation:

(5b) Does $K$ contain sentences that can serve as the explanans of an I-S explanation of $i$'s being $G$; and if so, what is the associated probability which the explanans confers on the explanandum sentence '$Gi$'?

The inductive probabilities referred to in the two questions are largely independent of each other. For example, as noted earlier, when an explanation of $i$'s being $G$ is sought, the sentence '$Gi$' is normally included in $K$. In that case, the probability of '$Gi$' on $K$ is 1; yet if $K$ contains sentences like the premisses of (4), which can serve to explain $i$'s being $G$, these sentences will confer upon '$Gi$' a probability that is less than 1. And that is quite reasonable; for *the point of an explanation is not to provide evidence for the occurrence of the explanandum phenomenon, but to exhibit it as nomically expectable.* And the probability attached to an I-S explanation is the probability of the conclusion relative to the explanatory premisses, not relative to the total class $K$. Thus, the requirement of total evidence simply does not apply to the determination of the probability associated with an I-S explanation, and the requirement of maximal specificity is *not* "a rough substitute for the requirement of total evidence."

As noted in my earlier articles on the subject, my conception of the maximal specificity condition was influenced by Reichenbach's rule of the narrowest reference class. I will therefore briefly indicate how I see the relation between that rule and the requirements of total evidence and of maximal specificity. Reichenbach proposed his rule as a method of assigning a probability, or a "weight," to what he called "a single case," such as recovery of an individual patient from his illness, or getting Heads as the result of the next flipping of a given coin. He held that "there exists only one legitimate concept of probability," namely, the statistical one, "which refers to classes," not to individual events; and that, therefore "the pseudo-concept of a probability of a single case must be replaced by a substitute constructed in terms of class probabilities" ([8], p. 375). This substitute notion of the weight to be assigned to the occurrence of a certain kind of event, say $G$, in a particular single case, say $i$, Reichenbach construed as the estimated statistical probability of $G$ in the "*narrowest*" reference class containing $i$ "*for which reliable statistics can be compiled*,"[6] i.e. in our parlance: for which $K$ includes reliable statistical information.

Reichenbach's rule then is intended to answer questions of type (5a). (Indeed, as far as I am aware, he never explicitly examined the logic of explanations based on probabilistic laws.) The rule may, in fact, be viewed as Reichenbach's version of the requirement of total evidence: it requires consideration of the total evidence and specifies what parts of it count as relevant to the weight of a single case, and how they are to be used in computing that weight.

**RMS**, on the other hand, pertains to questions of type (5b). Its function is not to assign a probability to '$Gi$', but to specify conditions under which two sentences in $K$—a law, '$p(G, F) = r$' and a singular sentence '$Fi$'—can serve to explain, relative to $K$, $i$'s being $G$. The necessary condition laid down by **RMS** is that for every subclass of $F$ to which $K$ assigns $i$—*and hence also for the narrowest of these, their intersection*, say, $S$—the class $K$ must contain a statement to the effect that within that subclass, the probability of $G$ equals $r$ (except when the probability in question is determined by the calculus of probability alone). The link to Reichenbach's principle lies in the fact that **RMS** has an implication concerning "the narrowest reference class." But this link, as it now appears, is substantively rather tenuous. For the narrowest reference class in the sense of Reichenbach's principle is by no means always the intersection $S$ just characterized; and while **RMS** normally requires, among other things, that $K$ contain a probability statement concerning $S$, Reichenbach's rule imposes no such condition on $K$ concerning $S$ or concerning the narrowest reference class in his own sense.

Just like Reichenbach's principle, **RMS** requires reference to the total evidence

[6] [8], p. 374 (emphasis in the original). Reichenbach acknowledged (p. 375) that his rule does not determine the weight in question univocally. It might be noted also that, even if the narrowest relevant reference class were always uniquely specifiable, the concept characterized by the rule would not have all the formal properties of a probability. For example, when two predicates, '$G_1$' and '$G_2$', logically exclude each other, the "probability," or weight, of '$G_1 i \vee G_2 i$' is not always the sum of the weights of '$G_1 i$' and '$G_2 i$'; for the narrowest reference classes available for determining the three weights may well not be identical, and this may result in a violation of the addition principle for probabilities. For a sympathetically critical discussion and revised statement of the rationale of Reichenbach's rule, see Salmon [9], pp. 90–94.

$K$, though for a different purpose, namely, in order to ensure that $K$ does not contain premisses suitable for an explanation conflicting with (4). The requirement of maximal specificity was intended to guarantee fulfillment of this latter condition by barring explanatory ambiguity. Recently, however, several writers have argued that in the form I have given it, the requirement falls short of its objective. I will now examine their reasons.

**3. The lawlikeness of explanatory probability statements and its significance for RMS.** W. C. Humphreys [6] has argued that **RMS** is so restrictive as to deprive virtually all the usual probabilistic-statistical laws of an explanatory role. In place of his illustration, I will give a strictly analogous one by reference to our earlier examples.

Let the content of $K$ amount to just what follows from the premisses of (3a) and (3c) and the further information that Jones is a member of a certain subclass $S'$ of $S \cdot I$ which has exactly four members. Then, Humphreys argues, by reason of this latter bit of information, of an almost always available and quite trivial sort, the argument (3c) violates **RMS** and is thus barred from explanatory status in $K$. He reasons as follows: The value of $p(-M, S')$, even though not explicitly specified by $K$, obviously must be one of the numbers 0, $1/4$, $1/2$, $3/4$, or 1 because according to $K$, the class $S'$ has just four elements. Now, according to **RMS**, (3c) qualifies as an I-S explanation relative to $K$ only if either (a) $K$ implies that the probability of $-M$ in $S'$ is the same as in $S \cdot I$, namely, .95, or (b) the probability $p(-M, S')$ is determined by the mathematical theory of probability alone. Humphreys does not explicitly consider the latter possibility, although his five "obvious" probability values are presumably determined by purely mathematical-combinatorial considerations. But he rightly points out that condition (a) surely is not met, and he concludes that therefore (3c) is ruled out as an I-S explanation. Similarly, he reasons, virtually any argument of the form (4) that would normally count as an I-S explanation can be disqualified on the basis of **RMS** by showing, and noting in $K$, that the individual case $i$ belongs to some small finite subclass of the reference class $F$ mentioned in (4).

But Humphreys' counterexamples can be barred by reference to the proviso, mentioned above, that the statistical probability statements on which probabilistic explanations can be based must be lawlike, must have the character of potential laws.[7] I will argue that the predicates occurring in lawlike probability statements—let us call them nomic predicates—must meet certain conditions, which are violated in the examples constructed by Humphreys.

The first condition, which in one form or another has been suggested by several writers, was adumbrated in my observation that laws of the universal conditional form

(6a) $\quad (x)(Fx \supset Gx)$

and statistical laws of the form

(6b) $\quad p(G, F) = r$

[7] See the discussion of this point in [4], pp. 121–124 and [5], pp. 376–380.

"share an important feature, which is symptomatic of their nomological character: both make general claims concerning a class of cases that might be said to be potentially infinite" ([5], p. 377). This is vague, however. For a class is either finite or infinite, leaving no room for "potentially infinite" classes. The quoted remark should be understood as ruling out any reference class that is finite on purely logical grounds, i.e. as a consequence of the way in which it is characterized by the "reference predicate expression" that occupies the place of '$F$'. Accordingly, the expression '$Fx$' in (6a) may not be logically equivalent to '$x = a \vee x = b \vee x = c$'; for then (6a) would be logically equivalent to '$Ga \cdot Gb \cdot Gc$', and such a finite conjunction of singular sentences lacks the force—explanatory and otherwise—of a law.

The two predicate expressions in a lawlike probability statement of form (6b) are subject to the same requirement:

> (N1) In a lawlike sentence of form (6b), neither the reference predicate expression, which takes the place of '$F$', nor the outcome predicate expression, which takes the place of '$G$', must have an extension that is finite on purely logical grounds.[8]

Indeed, the two predicates either stand for properties of things that can have "indefinitely many" instances (such as having blue-eyed parents and being blue-eyed); or they stand for *kinds* of events that are conceived as "indefinitely repeatable," such as flipping of a penny, and the penny landing Heads up; or a ten-second irradiation of a phosphorescent screen with alpha-particles from a given source, and the occurrence of from six to eight scintillations on the screen; or infection of a person with malaria plasmodium, and the person's catching malaria.[9] The probability statement then asserts, briefly, that in increasingly long series of cases instantiating the reference predicate, the proportion of those having the specified outcome tends to come, and remain, as close as we please to $r$. This claim presupposes that the predicate expressions stand for kinds of objects or events that can be conceived of as having infinitely many instances—at least "in principle," i.e. without logical inconsistency. And this is what N1 requires.

Now, predicates like that characterizing Humphreys' four-membered reference class $S'$ clearly violate the requirement N1 and are therefore barred by it.

The probability statements adduced by Humphreys are disqualified for yet another

[8] More explicitly: Let us say that an open sentence $S$ in one free predicate variable $U$ is a logical finiteness condition if (i) $S$ contains no constants other than those of logic and set theory, and (ii) $S$ is satisfied only by predicate expressions with finite extensions. Condition N1 is meant to require that if the reference predicate or the outcome predicate is expanded in primitive terms, and then substituted for $U$ in any logical finiteness condition $S$, the result is not a truth of logic or set theory.

[9] The predicates here said to stand for kinds of events need not be construed, however, as applying to entities of a special kind, namely, individual events (like those envisaged by Donald Davidson in "Causal Relations," *The Journal of Philosophy*, vol. 64, 1967, pp. 691–703); they may be treated instead as two-place predicates that apply to individual objects at certain times or during certain time intervals. Thus, the repeatable kind of event, flipping of a penny, need not be represented by a predicate that is true of certain individual events, namely, those that are penny-flippings: it can be symbolized instead by a two-place predicate that applies to an object $x$ at time $t$ just in case $x$ is a penny that undergoes a flipping at $t$.

reason. Explanatory laws, whether of universal or of probabilistic-statistical form, must be empirical in character; and the statement that the probability of $-M$ in $S'$ must have one of the values 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 is not: it simply expresses the logical truth that these are all logically possible proportions of those among four individuals who may have the property $-M$. This shows, moreover, that the statement is not a statistical probability statement at all; it does not concern the long-run relative frequency of the outcome $-M$ as the result of some repeatable kind of event.

I now turn to some criticisms and constructive suggestions made by G. J. Massey[10] concerning the adequacy of my formulation of **RMS** and concerning my earlier construal of the requirement as a "rough substitute for the requirement of total evidence."

In regard to that construal, Massey argues that I have furnished "no unobjectionable total evidence requirement which [the] rough criterion might subserve as a rule of thumb." Presumably his point comes to this: the total evidence requirement as formulated by Carnap is not even roughly subserved by **RMS**, for reasons stated in section 2 above; and that indeed I have offered no other version of the total evidence requirement for whose satisfaction **RMS** might play the role of a rule of thumb. I entirely agree.

Massey then suggests that I should have treated **RMS** "as a *bona-fide* substitute for the defunct total evidence requirement." Here I resist the implication that the total evidence requirement is defunct. In its proper place, namely, in determining the credence rationally assignable to a statement, I think it does ideally apply: the rational credibility in question depends on *all* the evidence available in the given situation. Practically, of course, we rely, explicitly or tacitly, on various judgments of irrelevance to trim the evidence down to manageable size.[11] On the other hand, I agree with Massey in viewing **RMS** as a *bona-fide* rule in its own right; but for the reasons given in section 2, I consider that rule as pertaining strictly to the probabilistic explanation of empirical phenomena, not to appraisals of the credibility of their occurrence: and quite possibly this is what Massey has in mind, too.

Massey then questions the adequacy of my formulation of **RMS** on two grounds. His first objection is based on counterexamples very similar to those constructed independently by Humphreys. But he goes on to suggest, correctly, that I would presumably reject the counterexamples on the ground that they use non-lawlike probability statements.

Massey's second objection is to the effect that if an argument like (4) is to qualify as an explanation in $K$, **RMS** requires $K$ to contain "a wholly unreasonable number of statistical laws": for normally, many predicates besides '$F$' will be known—i.e. will be said in $K$—to apply to $i$; the conjunction of any one of these with '$F$' determines a subclass of $F$; and $K$ is required to contain laws stating the probability

[10] See [7]. My discussion is based on a draft of [7] which Professor Massey sent me in January, 1966.

[11] An example is discussed in [4], pp. 142–143. See also Carnap's remarks on this point: [1], p. 494.

of $G$ within each of these subclasses (with the exception stated in **RMS**). This complaint will be met in section 4 below by a modification of **RMS** which is less stringent in this respect.

Another objection here to be considered has been raised by R. Wójcicki,[12] who illustrates his point, in effect, by one of my earlier examples for the ambiguity of statistical explanation ([4], p. 126). He argues that while **RMS** may eliminate such ambiguity, it qualifies as explanatory certain arguments that no scientist would regard as such. He reasons as follows:[13] Let '$Px$' stand for '$x$ is a person with a streptococcal infection who has been treated with penicillin'; '$Rx$' for '$x$ recovers quickly'; '$Qx$' for '$x$ has the property $P$, but he also has a certain physiological characteristic whose presence makes quick recovery very unlikely'; finally, let '$j$' again be short for 'Jones'. Now suppose that $K$ contains just the following sentences and their consequences:

$$p(R, P) = .95$$
$$p(-R, Q) = .96$$
$$(x)(Qx \supset Px)$$
$$Pj$$

as well as the definition

$$Q^+x =_{\mathrm{df}} Qx \vee (x = j)$$

and consequently also the sentence

$$(x)(Q^+x \supset Px)$$

Since the class determined by '$Q^+$' has at most one element more than that determined by '$Q$', non-recovery will have the same probability in $Q^+$ as in $Q$, so that $K$ also contains the sentence

$$p(-R, Q^+) = .96$$

Hence, $K$ contains the premisses of the following two rival arguments:

$$\text{(7a)}\qquad \begin{array}{l} p(R, P) = .95 \\ Pj \\ \hline\hline Rj \end{array} \quad [.95]$$

$$\text{(7b)}\qquad \begin{array}{l} p(-R, Q^+) = .96 \\ Q^+j \\ \hline\hline -Rj \end{array} \quad [.96]$$

[12] In his review article [10], which was published in 1966. Dr. Wójcicki had presented the idea to me already in January, 1965. His article also contains interesting critical comments on my construal of deductive-nomological explanation; but these cannot be discussed here.

[13] I slightly tighten Wójcicki's formulation so as to make explicit the requisite relativization with respect to $K$; and, for clarity of statement, I specify definite quantitative probabilities instead of speaking of high probabilities or near-certainties, as does Wójcicki, and as I had done in [4], p. 126.

Of these, Wójcicki rightly points out, **RMS** qualifies the second rather than the first as explanatory since $K$ contains the information that $Q^+$ is a subclass of $P$. Undeniably, this is a counterintuitive and unreasonable verdict. The objection can be met, however, by arguing that '$Q^+$' does not qualify as a nomic predicate. For '$Q^+$' applies to a certain individual, namely $j$, on purely logical grounds: '$Q^+j$' is short for '$Qj \vee (j = j)$', which is a logical truth. And this violates a second condition which, I would suggest, must be met by nomic predicates in probabilistic laws: no such predicate must *demonstrably* apply to any particular individual, or:

(N2) No full sentence formed from a predicate in a lawlike probability statement and an (undefined) individual name is a logical truth.

In its application to probabilistic-statistical laws, N2 can be supported by this consideration: The predicate '$P$' in (7a) stands for an indefinitely repeatable kind of event: streptococcal infection of a person, followed by penicillin treatment of that person. Similarly for '$Q$': suppose, for example, that the anti-recovery factor is a grave allergy to penicillin; then the repeatable kind of event is a streptococcal infection, followed by penicillin treatment, of a person suffering from that kind of allergy. But there is no analogous way of construing '$Q^+$'; for what kind of event would '$x = j$' stand for? If this consideration is correct, then it just makes no sense to assign a statistical probability to non-recovery with respect to the reference class characterized by '$Q^+$'.

This consideration supports N2 specifically for the reference predicates of probabilistic laws. But I think it may properly be extended to all nomic predicates, in lawlike sentences of probabilistic and of universal character; for it reflects the conception that it is not individual events or objects as such, but individuals as bearers of general characteristics that can enter into nomic connections.

While in earlier articles on I-S explanation, I had stressed that the relevant probabilistic-statistical statements must be lawlike, I had not explicitly stated conditions like N1 and N2 for the predicates permissible in such statements. I am indebted to the critics I have mentioned for obliging me to consider this point more closely.

In introducing N1, we noted in passing that the predicate expression in the place of '$F$' in a universal law of form (6a) must not be logically equivalent to an expression of the form '$x = a_1 \vee x = a_2 \vee \ldots \vee x = a_n$', where '$a_1$', '$a_2$', etc. are individual names. Nevertheless, the extension of the reference predicate in question may well be finite in fact and may even be the null class. For example, Newton's law of gravitation logically implies various more specific laws, most of them never explicitly formulated. One of these concerns the case where the two bodies in question are solid copper spheres having masses of $10^7$ and $10^8$ grams, respectively; and it expresses the mutual attractive force they exert upon each other as a function of the distance between the centers of the spheres. Now the reference class thus characterized, the class of all pairs of such spheres, may well, as a matter of fact, be finite—quite possibly null. But this does not follow logically from its characterization, and the consequence in question has the character of a law: it can serve, for example, to support subjunctive conditional statements such as predictions as to what forces *would* come into play *if* two such spheres *were* to be produced. Thus, in a lawlike sentence of the form (6a), the reference predicate may in fact, but not on logical grounds alone, be coextensive with an expression of the form '$x = a_1 \vee x = a_2 \vee \ldots \vee x = a_n$'. Replacement of the reference predicate by that expression would then turn the lawlike sentence into another that has the same truth value but is no longer lawlike.[14] Similarly, while the reference predicate in a lawlike sentence of form (6a) must satisfy N2, it may in fact be coextensive with another predicate expression which violates that requirement; replacement of the former by the latter again yields a nonlawlike sentence of the same truth value.

Epitomizing these observations we might say that a lawlike sentence of universal, nonprobabilistic character is not about classes or about the extensions of the predicate expressions it contains, but about those classes or extensions *under certain descriptions.*

An analogous remark applies to lawlike sentences of probabilistic-statistical form. Take, for example, the sentence

(8) $p(H, C) = 1 - (\frac{1}{2})^{100{,}000}$

where '$Cx$' and '$Hx$' are short for '$x$ is a succession of 100,000 flippings of a regular coin', and '$x$ is a succession of coin flippings at least one of which yields Heads'. The reference class, i.e. the extension of '$Cx$', then contains all and only those events each of which consists of 100,000 successive flippings of a regular coin; and the total number of such events ever to occur may well be small or even zero. Suppose there are exactly three such events, $e_1$, $e_2$, and $e_3$. Then '$Cx$' is coextensive with '$x = e_1 \vee x = e_2 \vee x = e_3$', which I will abbreviate by '$C^*x$'. Replacement of '$C$' by '$C^*$' in the lawlike probability statement (8) yields an expression that violates N1 as well as N2 and that is, therefore, not lawlike. Indeed, it might be added that because of these violations, the expression is not a significant probability statement at all since its reference predicate does not characterize an indefinitely repeatable kind of event. Accordingly, we note:

> **Statistical probability statements, and in particular those which are lawlike, concern the long-run frequency of a specified characteristic within a *reference class under some particular description,* and the predicate expressions that can serve as such descriptions must satisfy conditions N1 and N2.**

It follows that, properly, the requirement of maximal specificity should not be formulated as a condition on reference classes and their subclasses, as is **RMS** above, but as a condition on certain predicate expressions. A version which meets this condition will be developed in the next section.

It should be noted that conditions N1 and N2, though presumably necessary,

[14] On this point, my conception of lawlikeness differs from that advanced by Goodman in his pioneering work on the subject. Goodman would presumably assign lawlike status to any sentence obtained by replacing a predicate in a lawlike sentence by a coextensive one. For he characterizes the predicates that occur in lawlike sentences in terms of their entrenchment and stresses that entrenchment carries over from one predicate expression to any coextensive one; so that, in effect, "not the word itself but the class it selects is what becomes entrenched, and to speak of the entrenchment of a predicate is to speak elliptically of the entrenchment of the extension of that predicate." ([3], p. 95; cf. also my remarks on this point in [5], p. 343.)

are not sufficient to ensure lawlikeness. It is possible, for example, to construct probability statements which, though satisfying the two conditions, appear to be analogues to Goodman's examples of generalizations that receive no confirmation from their instances (cf. [3], pp. 72–83). Statements of the form '$p(G, F) = r$' do not, to be sure, have individual instances in the sense in which Goodman speaks of instances of universal conditional sentences; but they can receive support or disconfirmation from pertinent statistical evidence, i.e. from findings concerning the frequency of $G$ in finite sets of events of the kind characterized by '$F$'. This consideration suggests that statistical probability statements analogous to Goodman's nonlawlike universal conditionals might now be constructed in the manner of the following example: Let us say that an event is a fleagaroo jump, or a $J$ for short, if either it is examined before January 1, 2000 and is a jump made by a flea or it is not so examined and is a jump made by a kangaroo. And let us say that a jump is short, or $S$, if the distance it spans is less than one foot. Consider now the sentence:

(9) $p(S, J) = .9$

It meets the conditions N1 and N2. Suppose now that we gather relevant statistical data, which—with the year 2000 still rather far in the future—would concern exclusively the distances covered by flea jumps; and suppose further that in the vast, and steadily growing, set of observed jumps, the proportion of short ones is, and remains, very close to .9. This would not tend to support the general claim made by (9) because of its implications for $J$'s examined after the twentieth century.

The expression (9), then, is a rather close analogue, for statistical probability statements, to Goodman's nonlawlike universal conditionals. Doubtless, we would not qualify (9) as lawlike. But we might even question whether it constitutes a significant probability statement, albeit a presumably false one. A profitable assessment of this issue would require a much more thorough analysis than has here been suggested of the "meaning" of statistical probability statements, and I will not pursue this question further in the present context.

**4. A Revision of RMS.** The requirement of maximal specificity was meant to preclude the possibility of conflicting I-S explanations, and in [5] (p. 401) I offered an argument purporting to prove that, stated in a form essentially tantamount to **RMS** above, it does have the desired effect. But since then, my colleague, Dr. Richard Grandy, has pointed out to me[15] that my argument is fallacious, and has illustrated this by the following counterexample:

Let $K$ contain the premisses of the following two arguments (and, of course, their logical consequences)

(10a)
$$\frac{\begin{array}{c} p(G, F \vee G) = .9 \\ Fb \vee Gb \end{array}}{Gb} \quad [.9]$$

[15] In an unpublished note he wrote in February, 1966, as a graduate student at Princeton.

$$(10b)\quad \begin{array}{l} p(-G, -F \vee G) = .9 \\ Fb \vee Gb \\ \hline\hline -Gb \end{array} \quad [.9]$$

The first of these arguments satisfies **RMS**. For the only subclasses of $F \vee G$ which $K$ tells us contain $b$ are $(F \vee G)(-F \vee G)$, which is $G$; and $(F \vee G)G$, which again is $G$. But $p(G, G) = 1$ by virtue of the theory of probability alone. Thus, (10a) fulfills **RMS**. But an analogous argument shows that **RMS** is satisfied by (10b) as well. Thus, both of the two conflicting arguments qualify, under **RMS**, as I-S explanations relative to $K$.

Grandy's example raises a difficulty also for the alternative to **RMS** proposed by Humphreys [6] under the name "The rule of complete evidence." Humphreys' own formulation is somewhat vague because, among other things, it fails to observe the distinction between what is the case and what is known or believed, i.e. what is asserted by $K$ to be the case. But the rule he intended seems to come to this: Let $K$ contain statements to the effect that the individual case $n$ belongs to the classes $C_1, C_2, \ldots, C_m$, and that the probabilities of $W$ relative to these are $p(W, C_i) = r_i$ $(i = 1, 2, \ldots, m)$; and let the $C_i$ be *all* the classes for which $K$ provides this twofold information. Then a probabilistic explanation of $n$ being $W$ (or of $n$ being $-W$) is possible in $K$ if and only if $K$ contains a law specifying the probability of $W$ with respect to the intersection, $C$, of all the $C_i$; and it is on this law, '$p(W, C) = r$', that the explanation must be based.

This rule has one clear advantage over **RMS**: it makes much less stringent demands concerning the probabilistic-statistical laws that $K$ is required to contain, and it thus goes a long way towards meeting the objection, mentioned above, that Massey has raised against **RMS** on this score. But when applied to Grandy's example, Humphreys' rule implies that with respect to the given class $K$, the following is a proper explanatory argument:

$$\begin{array}{l} p(G, G) = 1 \\ Gb \\ \hline\hline Gb \end{array} \quad [1]$$

And this surely is unacceptable.

I will now suggest a modified version, **RMS***, of the requirement of maximal specificity, which avoids the pitfalls we have considered. First, some auxiliary concepts and observations.

Let '$F_1$' and '$F_2$' be short for two one-place predicate expressions. Then the first will be said to *entail* the second if '$(x)(F_1x \supset F_2x)$' is logically true; and it will be called *stronger* than the second if it entails, but is not entailed by, the second. Two predicate expressions that entail each other will be called *logically equivalent*. If '$F$' entails '$G$' or '$-G$', then, by the theory of probability alone, $p(G, F)$ equals 1 or 0, respectively; and, as noted at the end of section 1, the 'unless'-clause in **RMS** is meant to refer to just those probability statements in which the reference predicate thus entails the "outcome predicate" or its negate.

'$F_1$' will be called an *i-predicate in K* if $K$ contains '$F_1 i$'; and '$F_1$' will be said to be *statistically relevant* to '$Gi$' in $K$ if (1) '$F_1$' is an $i$-predicate that entails neither '$G$' nor '$-G$' and (2) $K$ contains a lawlike sentence '$p(G, F_1) = r$' specifying the probability of '$G$' in the reference class characterized by '$F_1$'.

Now, one essential feature of **RMS*** will be this: **RMS** imposes conditions on all classes to which $K$ assigns $i$; or, more accurately, on all $i$-predicates (save those entailing '$G$' or '$-G$') by which those classes are characterized in $K$. In **RMS***, only those $i$-predicates which are statistically relevant to '$Gi$' will be subject to similar conditions. In this respect, **RMS*** is analogous to Humphreys' rule; and like the latter, it is much less demanding than **RMS** in regard to the probabilistic laws that $K$ is required to contain.

Another modification of **RMS** is intended to avoid the difficulty noted by Grandy. Let us call a predicate expression, say '$M$', *a maximally specific predicate related to* '$Gi$' in $K$ if (1) '$M$' is logically equivalent to a conjunction of predicates that are statistically relevant to '$Gi$' in $K$; (2) '$M$' entails neither '$G$' nor '$-G$'; (3) no predicate expression stronger than '$M$' satisfies (1) and (2); i.e. if '$M$' is conjoined with a predicate that is statistically relevant to '$Gi$' in $K$, the resulting expression entails '$G$' or '$-G$', or else it is just equivalent to '$M$'. Every such most specific predicate is evidently an $i$-predicate in $K$.

The proposed modification of the requirement of maximal specificity can now be stated as follows: An argument

$$\text{(11)} \qquad \begin{array}{l} p(G, F) = r \\ Fi \\ \hline\hline Gi \end{array} \quad [r]$$

where $r$ is close to 1 and all constituent statements are contained in $K$, qualifies as an I-S explanation relative to $K$ only if the following condition is met:

(**RMS***) For any predicate, say '$M$', which either (a) is a maximally specific predicate related to '$Gi$' in $K$ or (b) is stronger than '$F$' and statistically relevant to '$Gi$' in $K$, the class $K$ contains a corresponding probability statement, '$p(G, M) = r$', where, as in (11), $r = p(G, F)$.

To illustrate: let '$F$', '$G$', '$H$', '$J$', '$N$' be logically independent predicate constants, and let $K$ contain just the following sentences and their consequences: '$Fi$', '$Gi$', '$Hi$', '$Ji$', '$Ni$'; '$p(G, F) = .95$', '$p(G, F \cdot H \cdot J) = .95$'. Then the $i$-predicates in $K$ are the five predicate constants just mentioned and all the predicate expressions that they singly or jointly entail. The predicates statistically relevant to '$Gi$' in $K$ are '$F$' and '$F \cdot H \cdot J$'; and, apart from logically equivalent versions, the last of these is the only maximally specific $i$-predicate related to '$Gi$'. Hence, if the argument (11), with $r = .95$, is to qualify as an explanation in $K$, **RMS*** requires $K$ to contain the sentence '$p(G, F \cdot H \cdot J) = .95$': and this condition is satisfied. **RMS*** is satisfied also by an alternative argument with '$Gi$' as the explanandum: its explanans consists of the sentences '$Fi \cdot Hi \cdot Ji$' and '$p(G, F \cdot H \cdot J) = .95$'.

Now let $K'$ be the class obtained by adjoining to $K$ the sentence '$p(G, F \cdot H) = .1$' (and by closing this set under the relation of logical consequence). Then there are the same $i$-predicates in $K'$ as in $K$, and just one more of them is statistically relevant to '$Gi$', namely, '$F \cdot H$'; again, '$F \cdot H \cdot J$' is the only maximally specific $i$-predicate related to '$Gi$'. Thus, condition (a) of **RMS*** is satisfied for (11) in $K'$. But (11) should not have explanatory status in $K'$[16]; for the information it adduces, that $i$ is $F$ and that the probability for an $F$ to be a $G$ is high, cannot count as explaining $i$'s being $G$ since $K'$ tells us further that $i$ belongs to a subclass, $F \cdot H$, of $F$ for whose members the probability of being $G$ is very small. But by reason of containing this latter probability statement, $K'$ violates clause (b) of **RMS***, and (11) is thus denied explanatory status in $K'$. Relative to $K'$, there is essentially just one explanatory argument with '$Gi$' as its conclusion: its explanans sentences are '$p(G, F \cdot H \cdot J) = .95$' and '$Fi \cdot Hi \cdot Ji$.'

Note that **RMS**, in contrast to **RMS***, would have barred (11) from explanatory status in the class $K$ of our first illustration; for it requires $K$ to contain a number of additional statements, assigning the probability .95 to $G$ with respect to the reference classes determined by '$F \cdot H$,' '$F \cdot J$,' '$F \cdot N$,' '$F \cdot H \cdot J \cdot N$,' '$F \cdot (H \vee N)$,' and so forth.

But while **RMS*** is less demanding than **RMS** in this respect, it is more exacting in another; for its clause (a) imposes a condition on *all* maximally specific predicates related to '$Gi$', and not only on those that entail '$F$'.

It is by reason of this stricter condition that **RMS*** escapes the pitfall noted by Grandy. In the class $K$ of Grandy's illustration, the $b$-predicates are '$F \vee G$' and '$-F \vee G$' and all the predicate expressions they entail. The two expressions just cited are the only ones that are statistically relevant to '$Gb$' in $K$; and each of them is also a maximally specific predicate related to '$Gb$'. Thus, if (10a) is to be an explanation, **RMS*** requires that $K$ contain the statement '$p(G, -F \vee G) = .9$'; and this condition is not satisfied.

**RMS*** quite generally precludes the possibility of conflicting explanations. For suppose that $K$ contains the premisses of the arguments

$$\begin{array}{l} p(G, F_1) = r_1 \\ F_1 i \\ \hline\hline Gi \end{array} \quad [r_1]$$

and

$$\begin{array}{l} p(G, F_2) = r_2 \\ F_2 i \\ \hline\hline Gi \end{array} \quad [r_2]$$

Let '$F$' be short for one of the maximally specific predicates related to '$Gi$' in $K$. Then *both* of the arguments qualify as explanations in $K$ only if $K$ contains statements to the effect that $p(G, F) = r_1$ and $p(G, F) = r_2$; but then, $r_1 = r_2$, and there

[16] I am much indebted to Dr. Richard Grandy, who pointed out to me the need to meet situations of the kind here under discussion, and who suggested that this might be done by means of clause (b) in **RMS***.

is no conflict. Thus, it appears that in the version **RMS***, the requirement of maximal specificity serves the purpose for which it was intended.

In conclusion, I summarize the construal here proposed for the concept of probabilistic, or I-S, explanation: An argument of the form

$$\begin{array}{l} p(G, F) = r \\ \quad Fi \\ \hline\hline \quad Gi \end{array} \qquad [r]$$

is a probabilistic explanation of basic form relative to a class $K$ (of the general kind characterized in section 1) if and only if

(1) $K$ contains the explanans and the explanandum sentences of the argument.
(2) $r$ is close to 1.
(3) The probability statement in the explanans is lawlike.
(4) The requirement of maximal specificity as expressed in **RMS*** is satisfied.

For the predicate expressions permissible in lawlike probability statements, the conditions N1 and N2 were proposed as necessary, but presumably not sufficient.

REFERENCES

[1] Carnap, Rudolf, *Logical Foundations of Probability*, University of Chicago Press, 1950.
[2] Glass, Bentley, *Science and Ethical Values*, The University of North Carolina Press, Chapel Hill, 1965.
[3] Goodman, Nelson, *Fact, Fiction, and Forecast* (Second edition), The Bobbs-Merrill Company, Inc., Indianapolis, 1965.
[4] Hempel, C. G., "Deductive-Nomological *vs.* Statistical Explanation," in *Minnesota Studies in the Philosophy of Science*, vol. III (eds. H. Feigl and G. Maxwell), University of Minnesota Press, Minneapolis, 1962, pp. 98–169.
[5] Hempel, C. G., "Aspects of Scientific Explanation," in *Aspects of Scientific Explanation and other Essays in the Philosophy of Science*, The Free Press, New York, 1965, pp. 331–496.
[6] Humphreys, W. C., "Statistical Ambiguity and Maximal Specificity," *Philosophy of Science*, vol. 35, No. 2, 1968, pp. 112–115.
[7] Massey, Gerald J., "Hempel's Criterion of Maximal Specificity," *Philosophical Studies*, vol. XIX, No. 3, 1968.
[8] Reichenbach, Hans, *The Theory of Probability*, The University of California Press, Berkeley and Los Angeles, 1949.
[9] Salmon, Wesley C., *The Foundations of Scientific Inference*, The University of Pittsburgh Press, Pittsburgh, 1967.
[10] Wójcicki, Ryszard. "Filozofia Nauki W Minnesota Studies," *Studia Filozoficzne* for 1966, pp. 143–154. This is a review article on *Minnesota Studies in the Philosophy of Science*, vol. III (eds. H. Feigl and G. Maxwell), University of Minnesota Press, Minneapolis, 1962.